Thomas Piketty's Economics: Modeling Wealth and Wealth inequality

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Contents

The present study supplements the two articles (one article in two parts) we already devoted to Piketty's theses in *Capital in the Twenty-First Century*[1], concerning the historical dynamics of wealth accumulation and inequality in capitalism[2].

Both in the presentation of theoretical frameworks (notably in the exposition of the "laws" of capitalism) and factual interpretation (for example, the analysis of the comeback of capital during the most recent decades), Piketty tends to approach the components of wealth jointly. Section 1 provides of more detailed exposition of our view that no such common treatment can be given of the various components of wealth. The main distinction is between capital as used by enterprises in production (fixed capital), and stocks of wealth resulting from the capitalization of flows of incomes, such as the value of land in agriculture, mining, or in the housing sector. (As in the rest of this study, by "value", we mean the total price of such components.) Contrary to what could be expected, the consideration of land did not become obsolete as a result of the comparative decline of agriculture, due to the rising importance of rent in the building sector (housing and offices). Besides land, treatments are also given of residential investment for the use of their owners, and the favorable situation of the financial sector in neoliberalism. In sharp constrast with the line of argument in *Capital in the Twenty-First Century*, we believe it is crucial to begin with the consideration of capital as used in production and, gradually, extend the field of analysis to incorporate the features of the various components, first separately and, then, in combination. The main contention is that the ensuing adjustments of theoretical frameworks allow for more adequate treatments of both the historical variations of the trends of wealth stocks, on the one hand, and the distinct effects of possible shocks impacting the trajectories of these variables, on the other hand.

Section 2 is devoted to a second generalization of the basic fixed capital model, concerning wealth inequality. Instead of various components of wealth, distinct groups of economic agents are introduced. The criticism we made in our previous article of Piketty's analysis of wealth inequality is first recalled, but the main object is the introduction of three models in which various mechanisms are considered susceptible of accounting for possible trends toward a rising concentration of wealth at the top of social hierarchies in capitalism. The conditions for the prevalence of such tendencies are finally discussed.

We already discussed the trend toward rising income inequality (mostly concerning wages) in the United States and the United Kingdom in our first article. Despite its crucial importance, we do not return to this issue here. The reason is that no analytical framework is defined by Piketty to account for this aspect of inequality, fundamentally a political development.

1. T. Piketty. *Capital in the Twenty-First Century.* The Belknap Press of Harvard University Press, Cambridge, MA, 2014

2. G. Duménil and D. Lévy. The Economics and Politics of Thomas Piketty's Theses. I - Critical Analysis. Paris, 2014a; G. Duménil and D. Lévy. The Economics and Politics of Thomas Piketty's Theses. II - An Alternative Reading of the History of Capitalism. Paris, 2014b

1 The Fluctuations of the Ratio of Wealth to National Income

In Piketty's analysis, the terms "wealth" and "capital" are used as synonymous. Land, for example, defines a first category of capital in this broad sense, though land is not produced. Correlatively, "capital income" includes rents. National wealth or capital is the sum of four components: (1) *land*; (2) *housing*; (3) *other domestic capitals*; and (4) *net foreign capitals*. (When securities are issued, the values of the various components are measured by the prices of the securities.) Piketty expresses wealth or capital stocks as percentages of national income (or output for brevity), thus accounting for the overall growth of the economies or inflation. This procedure is convenient but can, sometimes, be misleading.

The notion of national wealth is tricky. First, a process of aggregation is required in which double counting must be avoided. Notably it is not possible to count simultaneously the value of the fixed capital of enterprises and their net worth. A problem of valuation is also involved since stocks are purchased and sold on the stock market at prices that differ from the accounting value of corporations. (The ratio between the two measures is the well-known Tobin's q.) Second, assets and liabilities reciprocally held by various agents cancel out. For example, if both the private economy and government are introduced simultaneously (abstracting from the rest of the world), the debt of the government is an asset of other economic agents and disappears in the total. The same is true of money if both nonfinancial and financial agents are jointly considered. National wealth does not boil down, however, to the sum of the fixed capital (and inventories) of enterprises and housing, due to the existence of non-produced stocks of wealth such as land.

1.1 Accumulating Fixed Capital

This section is devoted to the core model, in which the fixed capital of enterprises is the single form of wealth. Two basic mechanisms are involved: (1) The capital stock at a given point in time is the outcome of a previous process of accumulation; and (2) The existing stock in one period is used for production. This common-sense approach is at the center of both classical-Marxian (later "classical") and neoclassical models.

In such models of accumulation, abstraction is made of business-cycle fluctuations. Output is equal to demand – equivalently, there is no inventories – at a level of activity that ensures the normal utilization of production capacities. Output is the sum of the two components of demand, namely consumption and investment: $Y = C + I$. Savings are the excess of consumption over income: $S = Y - C$. One, consequently, has:

$$\text{Savings} \quad = \quad \text{Investment}$$

The following sections contrast this approach to basic economic mechanisms and Piketty's substitute reading of formally identical relationships, though approached from a quite distinct angle.

1.1.1 The Classical Framework: Capital-Output-Savings-Investment-Capital

We consider here a straightforward classical model of accumulation in the strict sense, in which, besides the above assumptions, the situation of income distribution and the technique of production are given. Obviously, these distributional and technical conditions are susceptible of historical variations, but the hypothesis underlying the definition of such frameworks is that the mechanisms under investigation in the model are not fundamentally unsettled by such variations, at least qualitatively (that is, concerning the identification of the chain of events and relations of causation).

Four exogenous variables are typically defined: (1) the real wage, w; (2) the output/labor ratio, that is, the productivity of labor, $P_L = Y/L$; (3) the output/capital ratio or the productivity of capital, $P_K = Y/K$; (4) the saving rate on national income, $s = S/Y$.[3] Actually, only the ratio of the real wage to labor productivity $w/P_L = \omega$, that is, the share of labor in total income, is involved in the derivations below. Thus, the number of exogenous (given) parameters is only three: ω, P_K and s.

All other variables can be derived from the above. The share of profits is $\pi = 1 - \omega$, and the rate of return (or rate of profit), $r = \Pi/K = (\Pi/Y)(Y/K) = \pi P_K$. Since the addition to the stock of capital in one period, that is, the new investment, is equal to savings, the growth rate, g, of the capital stock is: $g = I/K = S/K = (S/Y)(Y/K) = sP_K$. Since the productivity of capital is given, g is also the growth rate of output. Beginning with a given stock of capital K_0, the capital stock and output in period t are:

$$K_t = K_0(1+g)^t \quad \text{and} \quad Y_t = P_K K_t = P_K K_0 (1+g)^t \quad \text{with} \quad g = sP_K \quad (1)$$

Not only the capital stock and output grow at the same rate g, also wages, profits, investment, and savings.

1.1.2 Piketty's Framework: From the Growth Rate to the Technique of Production

Relationships similar to the above are implied in Piketty's approach, though the notation is slightly different. The share of profits is denoted α instead of π in the above model; the ratio of capital to output is β, that is the inverse of capital productivity: $\beta = 1/P_K$. Thus, Piketty's first law of capitalism, $\alpha = r\beta$, the equivalent of $\pi = r/P_K$, matches the classical relationship $r = \pi P_K$. (The profit rate, or rate of return on capital, is the product of the share of profits by the productivity of capital.) In a similar manner, with the common notation s, Piketty's second law of capitalism, $\beta = s/g$, is the equivalent of $1/P_K = s/g$, nothing else than the classical

3. The two productivities should be denoted as "apparent" productivities.

relationship $g = sP_K$ above.[4] To sum up:

Piketty's first law:	$\alpha = r/\beta$	Classical formal equivalent:	$r = \pi P_K$.
Piketty's second law:	$\beta = s/g$	Classical formal equivalent:	$g = sP_K$.

We will later use the ensuing relationship that directly follows from Piketty's two laws:

$$\alpha = \frac{sr}{g} \quad \text{or, in our notation,} \quad \pi = \frac{sr}{g} \tag{2}$$

One should notice that the share of profits is smaller than 1: $\pi < 1$. Since $\pi = rs/g$, the value of the growth rate must satisfy the condition $g > sr$. This is equivalent to the assumption that total savings are larger than the savings proceeding from profits.

The choices made in the expression of exogenous parameters are obviously not neutral, as the directions of causation are interpreted differently. Piketty's choice of exogenous parameters differs from the options underlying the classical approach. The first law being an identity, no relation of causation is implied. Conversely, a causation is implied in the second law. Instead of the classical sequence going from an exogenous situation of technology (a given value of P_K) to the rate of growth of output, $g = sP_K$, Piketty explains β by g and s, as in $\beta = s/g$ or $P_K = g/s$, that is, the technique of production, by an exogenous growth rate.

1.1.3 Does Solow's Model Save Piketty's Direction of Causation?

Piketty's analytical scheme going from the exogenous growth rate of the population to the growth rate of output and, finally, to the technique of production is evocative of models in which the growth rate of the population is treated as an exogenous parameter as, notably, in Solow's accumulation model to which Piketty refers. The assumptions are: (1) a technology (the set of available techniques of production) as described by a production function, the basis of the minimization of costs by enterprises; (2) the saving rate; and (3) the growth rate of the population available for production. The model shows that one particular situation of distribution would ensure, if it prevailed, that the growth rate of the population be equal to the growth rate of employment. (Formally, an equilibrium exists.) In this situation, the technique of production is determined (by the minimization of costs).

Compared to the classical framework in the previous section, the basic mechanisms governing accumulation are conserved, but two important transformations occur: (1) The technique of production is determined endogenously (on the basis of the production function), that is, P_K is an endogenous variable; (2) The choice of other exogenous parameters is altered, specifically

4. The expression $g = sr$, as in the model of Section 1.1.5, is more familiar. The difference is due to the alternative definitions of the saving rate, either in relation to total income or profits.

the growth rate is now given and the situation of distribution determined endogenously. This second difference is, by far, the most important, testifying to the existence of two radically different viewpoints concerning basic economic mechanisms in capitalism.[5]

Piketty, whose main variable is the inverse of the productivity of capital, enters himself into this discussion in his Chapter 6, considering a Cobb-Douglas production function with constant returns to scale, $Y = cK^{\alpha}L^{1-\alpha}$. Two results follow:

1. The productivity of capital is determined as $P_K = g/s$, that is, Piketty's second law $\beta = s/g$.
2. The model also determines the situation of distribution. As is well known, one has: $\pi = \alpha$ (a parameter in the production function), and the rate of return is $r = \pi P_K = \alpha s/g$.

Two features of this framework are noteworthy: (1) The rate of return is an endogenous variable; and (2) The rate of return and the growth rate are linked by an inverse relationship. These two properties are overlooked in *Capital in the Twenty-First Century*.

As recalled above, Solow's model shows that, in the framework defined by his assumptions, one situation of distribution exists in which the population and employment grow at the same rate. (One can recognize the traditional line of argument – a "flexible" wage rate ensures full employment.) The implications in terms of factual lines of causation are very questionable. Do the growth rates of output (the image of the growth rate of the population) determine the situation of distribution (the profit share and the real wage)? Can we take the coincidence between low growth rates and large rates of return seriously? Our answer is clearly negative, and this explains why we follow neither the straightforward implications of Solow's model – thus, used without detour in the interpretation of actual trends – nor Piketty's line of argument.

At a rather general level of analysis, one basic difference between Solow's framework and Piketty's analysis must be emphasized. Production (the capital used for production and the technique of production) is clearly central stage in Solow's model, contrary to what is observed in Piketty's framework. More technically, it is not even clear that Piketty really accepts the basic properties of Solow's model in which the equilibrium rate of return is an endogenous variable ($r = \alpha g/s$). Reading *Capital in the Twenty-First Century*, it often seems that the line of reasoning is based on the opposite viewpoint, namely that r is an exogenous parameter.

5. The contention that the situation of distribution has an impact on the technique of production (as in the first statement), as well as the acknowledgement of the reciprocal effect, is important in Marx's analysis (in the framework of the "Law of capitalist accumulation" at the end of Volume I of *Capital*). Marx's framework is, however, straightforwardly a disequilibrium framework, as the forces pushing capitalists to adopt more sophisticated techniques of production are manifested during the new wave of investment, in the wake of the occurrence of the recession and the ensuing wave of unemployment.

1.1.4 The Layers of Accumulated Savings

Piketty's second law of capitalism, $\beta = s/g$, is derived from a model that straightforwardly expresses the formation of the capital stock by the sequence of savings, accumulated during the previous periods:

$$K_t = \sum_{\tau<t} S_\tau$$

The summation is made to $\tau < t$, and not $\tau \leq t$, since the capital stock outstanding in one period is the outcome of previous accumulation. With the assumptions of a given saving rate and given growth rate, the total capital stock in period t can be easily derived:

$$K_t = \sum_{\tau<t} S_\tau = \sum_{\tau<t} (sY_\tau) = sY_0 \sum_{\tau<t} (1+g)^\tau = sY_0 \frac{(1+g)^t}{g} = \frac{s}{g} Y_t$$

The value of β in period t follows:

$$\beta_t = \frac{K_t}{Y_t} = \frac{s}{g} \tag{3}$$

A first important result is that, in this model in which wealth is limited to the fixed capital of enterprises, the ratio, $\beta_t = s/g$, of capital to output is a constant, that is, β_t is always equal to its equilibrium value. Only in a more complex model, β_t could differ from its equilibrium value (Section 1.2.1).

Piketty does not enter into the logics of capital accumulation for production. Conversely, these mechanisms are center stage in the framework of our Section 1.1.1, where capital is used for production with a given technology. In this context, the growth rate cannot be treated as an exogenous variable, but is determined by the saving rate and the technique of production: $g = sP_K$. As could be expected, importing this value of g into equation 3, it appears that the equation boils down to: $K_t/Y_t = s/(sP_K)$ or $P_K = Y_t/K_t$, that is, the definition of P_K.

1.1.5 Saving Exclusively from Profits

It is also possible to assume that only capitalists save or, equivalently, that there is no saving on wages (as in Kaleckian models). The growth rate is determined as in the well-known Cambridge equation: $g = sr$. (With $s < 1$, one has $g < r$.) Equation 3 can, thus, be written $\beta = s\pi/g$. Substituting sr for g, the identity $\beta = 1/P_K$ is recovered, and no fundamental property is altered.

In this context, it appears even more obviously that the equation $g = sr$ links Piketty's three exogenous variables. Again, this finding shows that only two of these variables can be chosen autonomously.

1.2 Non-Production Investment and the Capitalization of Rent

This section relaxes the strict consideration of a single component of wealth, namely the fixed capital of enterprises, as in the previous section, in two possible directions: (1) the existence of a second physical component of wealth, the outcome of production but not used for production, such as "construction for the use of the owner"; and (2) other nonproduced components of wealth, whose value is derived from the capitalization of a flow of income (rents), such as land. The section introduces a second framework, in which securities are issued in the financing of segments of the financial system to which the access is restricted.

1.2.1 Construction for Use

A new term, L, accounting for residential investment must be added to the equation that connects supply to demand: $Y = C + I + L$. In this model, one may refer to investment in a broad sense as the sum $I + L$, and savings as $Y - C$. The two saving rates are $I/Y = s_I$ and $L/Y = s_L$. National wealth (total net assets), $A = K + H$, is the sum of two components, fixed capital, K, and the housing stock, H. Since only fixed capital is used for production, one has: $g = I/K = s_I P_K$.

In the previous model, where only one component of wealth was considered, the summation could be made from any earlier period of time. In the present model, the values, K_0 and H_0, of the two stocks must be specified in a given initial period. One has:

$$K_t = K_0 + \sum_{0 \leq \tau < t} (s_I Y_\tau) = K_0 + s_I Y_0 \sum_{0 \leq \tau < t} (1+g)^\tau \tag{4}$$

$$= K_0 + s_I Y_0 \frac{(1+g)^t - 1}{g} = K_0 + \frac{s_I}{g} (Y_t - Y_0) \tag{5}$$

$$= \left(K_0 - \frac{s_I}{g} Y_0 \right) + \frac{s_I}{g} Y_t \tag{6}$$

A similar equation can be written for housing:

$$H_t = \left(H_0 - \frac{s_L}{g} Y_0 \right) + \frac{s_L}{g} Y_t \tag{7}$$

Introducing the relationships $g = s_I P_K$ and $Y_0/K_0 = P_K$ into equation 6 one obtains:

$$K_t = \frac{s_I}{g} Y_t \tag{8}$$

Thus, the ratio, β_t, of wealth, $A = K + H$, to output can be determined:

$$\beta_t = \frac{A_t}{Y_T} = \frac{H_0 - \frac{s_L}{g} Y_0}{Y_t} + \frac{s}{g} = \frac{H_0 - \frac{s_L}{s_I} K_0}{Y_t} + \frac{s}{g} \quad \text{with} \quad s = s_I + s_L \tag{9}$$

Contrary to the result obtained in Section 1.1.4, β_t varies with time. As output grows, an asymptotic value, β, is reached:

$$\beta = \frac{s}{g}$$

In such an equilibrium, one has: $H/K = s_L/s_I$. Only if this relationship holds in the initial conditions, that is, if $H_0/K_0 = s_L/s_I$, β_t is constant, being always equal to its equilibrium value.

As in the previous models, this equilibrium value can be expressed as a function of the saving rate and the productivity of capital:

$$\beta = \frac{s}{s_I} \frac{1}{P_K} = \left(1 + \frac{s_L}{s_I}\right) \frac{1}{P_K}$$

A new term is added to the earlier formula for β, the expression of the accumulation of the second component of wealth.

Similar models could be built to account for governments' investments in infrastructure, the purchase of durable goods by households, or the like.

1.2.2 Non-Reproducible Ressources: Land

This section is devoted to the consideration of any category of non-reproducible resources from which rents are derived. Such components of wealth, typically land, play a central role in Piketty empirical-historical analysis, in particular in Europe. For example, during the 1880s in France, the value of agricultural land was still larger than the value of *other capitals*, without mentioning the fraction of the value of the component *housing* which should be imputed to land for construction (Figure 7 in the second part of our first article). The dramatic rise of the value of housing during the most recent decades – the main component of the comeback of capital – can also be pinned on land, as suggested in the framework of the present section instead of the reference to the decline of the growth rate as in Piketty's model s/g, abusively extended to the value of a component of wealth for which it cannot account.

In the present section, we assume that such resources are only used for production. Correlatively, rents are only paid by enterprises. We do not enter here into the complexity of a theory of rent and assume that a given share of total income is paid as rent (as in the case of profits). National income, $Y = W + \Pi + R$, is the sum of three categories of incomes: (1)

wages W; (2) ground rent R; and (3) profits $\Pi = Y - W - R$. As in the previous sections, the situation of distribution is given:

$$W = \omega Y, \quad R = \gamma Y, \quad \Pi = \pi Y \quad \text{with} \quad \omega + \gamma + \pi = 1$$

The only productive component of wealth is still the fixed capital of enterprises, with $Y = KP_K$, and the saving rate is s. (The relationship between Y and the use of the non-reproducible resource is not explicitly expressed, as the technique of production is given.) One has: $g = sP_K$. The price of one unit of capital good is constant and, for simplicity, set to 1.

The value of land is determined by the capitalization of the flow of rent, with a capitalization rate, i:

$$T = \frac{R}{i} = \frac{\gamma}{i} Y \tag{10}$$

With $A = K + T$, the ratio of the wealth stock to output follows:

$$\beta = \frac{A}{Y} = \frac{K}{Y} + \frac{T}{Y} = \frac{1}{P_K} + \frac{\gamma}{i} \tag{11}$$

The difficult issue is the determination of the capitalization rate i. It can be treated as a given exogenous parameter, for example an interest rate. This is the option adopted in what follows. Other assumptions could also be made, notably that i is equal to the rate of return on fixed capital, that is, $i = r$ (with $r = \pi P_K$). In this case, importing this value into the above expression of β, the following equation is obtained:

$$\beta = \left(1 + \frac{\gamma}{\pi}\right) \frac{1}{P_K}$$

Substituting s/g for $1/P_K$ in Equation 11, one obtains the following relationship similar to Piketty's model, though more general:

$$\beta = \frac{s}{g} + \frac{\gamma}{i} \tag{12}$$

The capital stock increases with the various layers of investment (equal to savings), as expressed in the first term s/g, that is, Piketty's ratio. The relative value of land (T/Y, as in Equation 10) decreases (or rises) over time due to the decrease (or rise) of the share of rent (γ), as in the second term, γ/i, in the previous equation.

1.2.3 Fixed Capital, Housing, and Land

In the model of Section 1.2.1, a new component of wealth was introduced besides enterprises' fixed capital, namely housing as capital resulting from production, though unproductive capital. In Section 1.2.2, the existence of a component of capital only used in production and whose value was derived from the capitalization of a rent such as land was considered, also in addition to fixed capital, the source of a flow of rent for landowners. The purpose of the present section is to combine these two frameworks, considering the three components of wealth jointly, but also adding that land is used in the housing sector ("supporting" buildings). Thus, three social classes are involved, namely wage earners, capitalists, and landowners. A new point is that the owners of buildings must pay a rent to landowners.

Output, $Y = KP_K$, is produced using the stock, K, of fixed capital of capitalists. Capitalists pay wages, W and a rent R^K, and they earn a profit, $\Pi = Y - W - R^K$. The three categories of incomes are expressed as shares of national income, namely $\omega = W/Y$, $\gamma^K = R^K/Y$ and $\pi = \Pi/Y$. The rent, R^H/Y, from the housing sector is paid out of the flows of income derived from production. Landowners earn the two components of rent, R^K and R^H: $R = R^K + R^H$. We also define: $\gamma^H = R^H/Y$ and $\gamma = R/Y$. One has: $\omega + \gamma^K + \pi = 1$ and $\gamma^K + \gamma^H = \gamma$.

Output is equal to demand, that is, the sum of the three components: (1) investment in fixed capital, I; (2) investment in construction, L; and (3) consumption, C. One has: $Y = I + L + C$. These components of output can also be expressed as ratios of national income, $s_I = I/Y$, $s_L = L/Y$, and $c = C/Y$, with $s_I + s_L + c = 1$. The flow of investment in fixed capital is added to the existing stock, K, and the investment in construction, is added to the existing stock of housing, H.

The same assumptions are made as in the two previous sections, notably that the value of land is the result of the capitalization of the flow of rent (using a constant parameter, i, as capitalization rate). Since only fixed capital is used for production, the growth rate is still $g = s_I P_K$. The following results are obtained:

1. The equilibrium value of the ratio of the two components of wealth resulting from production (housing stock/fixed capital stock) is: $H/K = s_L/s_I$.

2. In any period, the value of the ratio of the value of land to national income is: $T/Y = \gamma/i$.

3. With $s = s_I + s_L$, the equilibrium value of Piketty's ratio, β, of national wealth to national income is consequently:

$$\beta = \frac{K+H+T}{Y} = \frac{K+H}{K}\frac{K}{Y} + \frac{T}{Y} = \frac{s}{s_I}\frac{1}{P_K} + \frac{\gamma}{i} \tag{13}$$

Substituting g/s_I for P_K, the above relation can be expressed as a further generalization of Piketty second law $\beta = s/g$:

$$\beta = \frac{s}{g} + \frac{\gamma}{i}$$

The same expression is recovered as in the previous section.

1.2.4 Barriers to Entry in the Financial Sector

A central aspect of the classical analysis of competition in capitalism is the view that the allocation of capital among various sectors of the economy depending of comparative rates of return results in a tendency toward the equalization of rates of return in the various industries (among industries, not among enterprises, due to the various levels of efficiency). This tendency is checked in various circumstances, notably when barriers prohibit or limit the movements of capitals. Such barriers can result from the existence of specific technical, institutional, legal, or political settings.

The situations, thus, created are evocative of the analysis in the previous section, as the access to the ownership of land is obviously restricted, and the advantage for the owners of these sectors is typically described as the benefit of a rent. We believe an important field of application of this framework in neoliberalism is the existence of a segment of the financial system to which ordinary savers do not have access, for example, institutions specialized in the management of assets of rich families or hedge funds.

A model similar to the previous can be defined in a very simple framework. The segment of the financial sector is considered unproductive, though, through financial transactions, it garners a fraction of total income (at the expense of other sectors). Abstraction is made of the fixed capital used (such as offices). We assume that only the "industrial sector" pays fees and interests to the financial sector. National income, $Y = W + F + \Pi$, is the sum of three components, wages W, the flow of profits F in the financial sector, and standard profits Π as in the "industrial sector". As in the previous models, the shares of total income accruing to each category of beneficiaries are normalized by national income:

$$\omega = \frac{W}{Y}, \quad \varphi = \frac{F}{Y}, \quad \pi = \frac{\pi}{Y} \quad \text{with} \quad \omega + \pi + \varphi = 1$$

The value of the capital in the industrial sector is K (with the given price 1), and B is the value of the financial sector. As in the previous model, the flow of income in this sector can be capitalized using a capitalization rate, i:

$$\frac{B}{Y} = \frac{\varphi}{i}$$

With $A = K + B$ denoting the total wealth, the ratio β of this stock of wealth to national income can be determined:

$$\beta = \frac{A}{Y} = \frac{K}{Y} + \frac{B}{Y} = \frac{1}{P_K} + \frac{\varphi}{i} \tag{14}$$

Substituting s/g for $1/P_K$, a relationship follows, again a generalization of Piketty's framework:

$$\beta = \frac{s}{g} + \frac{\varphi}{i}$$

Much more theoretical and empirical research would be required to combine the analysis of the fonctions of the financial sector, the mechanisms involved, the access to superprofits, and the existence of components of wealth based neither on the stock of capital used in production nor the consideration of a non-reproducible resource, the straightforward object of the present section. Such frameworks are evocative of, respectively, Marx's analyses of banking capital, competition, and fictitious capital.

1.3 The Dynamics of the Ratio of Wealth to National Income: Shocks and Historical Trends

The present section discusses Piketty's use of his second law in the analysis of both the overall trend of the ratio of wealth to output and the effects of shocks on this ratio. The thesis put forward in *Capital in the Twenty-First Century* points to the existence of a secular trend interrupted by the triple shock of the two World Wars and the Great Depression:

1. Concerning trends, in Piketty's analysis, the variations of the exogenous parameter g determine those of β. (A low value of g results in an elevated value of β, and the richest fractions of the population become even richer.) Involved here are the theoretical foundations of the empirical test we performed in our earlier study (Figure 5) of the explanatory power of Piketty's model concerning the trends of wealth generated by the accumulation of savings in the United States during the post-World War II period (when the data are the most reliable). It appeared that the second law of capitalism does not measure up to empirical confrontation.
2. Concerning shocks, still following Piketty, the triple shock caused the dramatic decline of β during several decades after World War II. This decline was gradually offset during the most recent decades, in the comeback of capital.

In those respects, our main contention is that neither trends nor shocks can be studied without separating (as in the models in the previous sections) between the main components of wealth, namely the capital of enterprises and the components whose values are derived from the capitalization of a flow of income. The focus is on the treatment of these latter components.

Concerning trends, another set of probably important reasons accounting for the failure of the empirical test is briefly discussed in the first subsection below, namely the absence of consideration of the distinct speeds manifest in the variations of variables and parameters.

1.3.1 Sticky Exogenous Parameters?

Piketty's second law of capitalism, $\beta = s/g$, has been devised in order to account for secular trends, actually three centuries of history. It is obvious that both parameters s and g, cannot be considered constant over such periods of time.

From a theoretical viewpoint, there is no problem in defining a sequential model in which parameters are assumed given, while it is simultaneously well known that they can vary over time. An hypothesis is, however, necessarily made by Piketty concerning the dynamics of these parameters, namely that they vary slowly, but this assumption remains implicit. (We obviously abstract from possible short-term fluctuations as along the phases of the business cycle.) If such "stickiness" is not observed, in comparison with the velocity of the dynamics of endogenous variables, the reference to a long-term (secular) tendency is irrelevant. Actually, as is always the case within such dynamic frameworks, it would be necessary to separate between the two categories of variables, slow and fast variables, and account separately for the two dynamics, namely the drift of the slow variables involved in the definition of the long-term equilibrium, and the "gravitation" of the fast variable(s). As is well-known, in the study of these fast dynamics, it is possible to assume that the slow variables are given. Symmetrically, the long-term dynamics can be studied under the assumption that the fast variables have converged.

The question must therefore be raised of the consequences of the absence of empirical control of the distinct speeds of the variables and parameters in *Capital in the Twenty-First Century*. The test performed in our first study shows these consequences are dramatic. The analysis in the remainder of the present section abstracts, however, from these difficulties.

1.3.2 Exogenous Parameters and the Rise of the Ratio of Wealth to National Income

The rise of β is a crucial aspect of Piketty's analysis, either in the study of the variations of β itself, or in the study of wealth inequality. (The rise of $\beta = A/Y$ means that the wealth stock, A, grows faster than national income, Y.) Various expressions have been given of the equilibrium value of β in the previous sections depending on the model considered: (1) fixed capital only; (2) fixed capital and housing for the owner's use; (3) fixed capital and a component of wealth whose price is determined by the capitalization of a flow of income, notably as in the components *land* and *housing*; (4) a model in which three categories of assets are jointly considered; and (5) a model with a financial sector. As contended, in each instance, g cannot be treated as an exogenous variable, since $g = sP_K$ (or $g = s_I P_K$). Five distinct expressions are obtained:

$$\beta = \frac{1}{P_K} \qquad \beta = \left(1 + \frac{s_L}{s_I}\right)\frac{1}{P_K} \qquad \beta = \frac{1}{P_K} + \frac{\gamma}{i}$$

$$\beta = \frac{s}{s_I}\frac{1}{P_K} + \frac{\gamma}{i} \qquad \beta = \frac{1}{P_K}\frac{\varphi}{i}$$

These formulas show that the rise of β may follow from three distinct categories of variations of the exogenous variables:

1. *A decline of the productivity of capital ($P_K \searrow$).* In the second part of our previous article, we have shown to what confusions the neglect of this

relationship may lead as Piketty interprets the increase of β as a sign of enrichment on the part of the owners of capital.

2. *A rise of the share of rent in total income* ($\gamma \nearrow$). Our article also showed the importance of this mechanism in the explanation of the comeback of capital during recent decades, whose main manifestation was the rise of the ratio of the value of housing to national income (in Europe). This trend cannot be interpreted, as Piketty does, in relation to the slow growth rates that prevailed within European countries ($g \searrow$).

3. *A rise of savings for the financing of residential investment, compared to saving devoted to the accumulation of fixed capital for production* ($s_L/s_I \nearrow$). This relationship also emphasizes the importance of other intuitive determinants (rather than g) in the interpretation of the upward trend of the value of housing in Europe.

4. *Finance in neoliberalism*: A larger φ results in a larger value of the financial sector.

5. *A rise of i, the rate of capitalization.* Larger values of this rate result in lower values of the corresponding component of wealth and, therefore, of β (not in the rise of wealth).

The contention that the rate of return on wealth, to which our Section 2 is devoted, is larger than the growth rate ($r > g$) is the central argument to which Piketty resorts to account for the tendency toward rising wealth inequality in capitalism, the second of his great theses. In *Capital in the Twenty-First Century*, it is sometimes difficult, however, to distinguish, between this thesis and the discussion of the rise of the ratio of the wealth stock to national income (see, for example, the discussion page 26). It should be clear from the analysis above that the value of the rate of return, r, does not impact β.

1.3.3 Shocks

A broad variety of shocks may affect the ratio of wealth to national income. A first category of shocks can lead to the physical destruction of capital (fixed capital or housing). A component of wealth whose value is determined by the capitalization of a flow of income can also be devalued due to the sudden diminution of this flow (for example, as a result of the control of rents in the housing sector, that is, a shock on the share, γ, of rents in national income). (In this discussion, we assume that the two exogenous parameters, s and P_K, are constant.)

Concerning shocks, in sharp contrast with Piketty's analysis, the contention in the present section is that the effects of shocks on the ratio, β, of wealth to national income crucially depend on the components involved. As in the previous section, the results are subject to the choice of the relevant model in Section 1.2 is considered:

1. *Only fixed capital.* Since $\beta = 1/P_K$ and P_K is constant, β is not affected: A destruction of fixed capital correspondingly impacts output.

2. *Fixed capital and construction for use.* As shown in section 1.2.1, in an equilibrium, the ratio, $H/K = s_L/s_I$, of the two wealth stocks is given. A

destruction of either one or both components alters this ratio, and equilibrium is destroyed. The trajectory of β_t toward the new equilibrium is described by equation 9, setting K_0 and H_0 at the values of the two stocks after the shock.

3. *Fixed capital and land.* In the model considered, the devalorisation of land can only be the effect of a shock on γ, the share of the rent in total income. In the event of a permanent decline of this share, land is permanently devalued.

4. *Fixed capital, construction for use, and land.* The results of the two previous models must be combined.

5. *The financial sector.* A shock such as the imposition of neoliberalism (with economic, institutional, and political aspects) increased the share of national income going to the financial sector, correlatively, its value in the stock market.

1.3.4 Historical Trajectories

Confronted to the empirical failure of Piketty's explanatory scheme, in our earlier study, we substituted a simple alternative framework – "our model" – based on two observations: (1) In the United States, the ratio to national income of the sum of two among the components of wealth, *land* and *housing*, remained roughly constant throughout the period; and (2) As could be expected, the profile of variation of the third component of wealth *other domestic capitals* is directly evocative of the inverse of the productivity of capital in the country.[6] In order to check the relevance of this interpretation, we used the following simple model:

$$\beta = \mathrm{cst}_1 + \frac{\mathrm{cst}_2}{P_K} \tag{15}$$

The comparison, in Figure 5 of our first article, between the values of the ratio of wealth to national income and the series as reconstructed appeared quite satisfactory. The models in the previous sections shed some light on this computation. One will easily check that the empirical model in equation 15 directly matches the theoretical framework of Section 1.2.3, in which fixed capital, housing, and land are jointly considered. The formal pattern of the expression given of β (Equation 13) in this section directly matches the model above for $\mathrm{cst}_1 = \gamma/r$ and $\mathrm{cst}_2 = s/s_I$. Thus, the two features observed above seem to be the expressions of relatively stable situations of both income distribution, for the first constant term, and the division among the two components of investment in the United States.

6. The value of the capital used for production is necessarily an important componant of wealth. It must appear somewhere in Piketty's data, namely in the component *Other domestic capitals*. This component is supposed to be measured as in the stock market, at least for the value of the corporations traded in this market, what will necessarily create some distance between the replacement cost and market estimates. Piketty himself consider that Tobin's q tends, however, to fluctuate around 1.

A second field of application of the theoretical framework in the present study is the interpretation of the main aspect of Piketty's analysis of the comeback of capital, specifically the rise of β during the most recent decades in the United Kingdom and France. In those years, the importance of land is manifest in the housing sector (not in the component *land* itself), actually the main component involved in this comeback. The relationship $T/Y = \gamma/r$ suggests possible explanatory mechanisms accounting for the rise of the price of housing in Europe. As could be expected, the link is established in the model between this rise and the growing share of rent in national income (as in Figure 6.7 of *Capital in the Twenty-First Century*, for France), an effect of urbanization.

2 The Trends of Wealth Inequality

In addition to the variations of the ratio of wealth to national income, the second main object of Piketty's investigation is the explanation of wealth inequality (more than income inequality) and the discussion of its historical trends, one of the main contributions of the book. The two notions are tightly related in *Capital in the Twenty-First Century* as the comeback of capital during the most recent decades is associated with a new rise of wealth inequality. But such convergences can also be misleading and, as contended in our first article, a more specific analysis is required.

Even restricting the analysis to wealth, inequality is already a complex and multifaceted phenomenon. Differences could be considered between housing owned as homes or rented, or between stock shares in pensions funds and other forms of holding, and the like. But Piketty does not enter into such investigations.

2.1 Piketty's Approach

At the origin of Piketty's contention concerning the rising trend of wealth inequality is the observation that the rate of return on wealth, r, is larger than the growth rate, g, of the economy. Piketty acknowledges that capital income is also used for consumption. Consequently, in the determination of the growth rate of the wealth of capitalists, r must be multiplied by the saving rate on capital income.

Formally, there is a quite straightforward manner of questioing Piketty's reference to r/g. Taking account of the savings of rich people, the model cannot be defended in the simple framework in which the saving rate of this category of people is equal to the average saving rate in the entire economy. In this case, the relation $r > g$ becomes $sr > g$. The contradiction is that, as shown earlier, in Piketty's framework, the relationship $sr < g$ necessarily holds (Equation 2), that is, savings are larger than the savings proceeding from profits. (The case in which a saving rate of reach people is larger than the average saving rate is assumed is discussed in our Section 2.2.)

A brief quotation was made in our previous study concerning Piketty's alleged mechanism (Section *Two theses on the history of capitalism*). We return here to this passage, though more extensively:

> When the rate of return on capital significantly exceeds the growth rate of the economy (as it did through much of history until the nineteenth century and as is likely to be the case again in the twenty-first century), then it logically follows that inherited wealth grows faster than output and income. People with inherited wealth need save only a portion of their income from capital to see that capital grow more quickly than the economy as a whole. Under such conditions, it is almost inevitable that inherited wealth will dominate wealth amassed from a lifetime's labor by a wide margin, and the concentration of capital will attain extremely high levels – ··· (p. 26)

(For simplicity, the discussion here can be conducted in a framework in which only the components of wealth resulting from production are considered, as in Section 1.2.1.) A difficulty in the interpretation of this analysis is that Piketty moves suddenly from one time frame to another in two successive sentences:

1. The statement in the first sentence points to a property inherent in a secular trajectory. In this context, all variables grow at the same rate, notably the relationship $g = \rho(A)$ necessary holds. Piketty's statement must be rejected (Section 1.1.1).
2. The time frame supporting the reasoning in the second sentence is one period (a lifetime). The addition of the capital amassed in one period to the existing stock of capital in a subsequent period is not considered (and so on, period after period, as in a sequential model). Piketty's viewpoint can only be defended in such a one-period framework. Denoting K the capital at the beginning of the period, and W the total wage earned during the period, "the wealth amassed from a lifetime's labor" is $s^W W$. Assuming for simplicity that all economic agents involved have the same saving rate, that is, $s^K = s^W = s$, the ratio between the two stocks of wealth is: [7]

$$\frac{sW}{K} = \frac{sY}{K} - \frac{s\Pi}{K} = sP_K - sr = g - sr$$

Consequently, the ratio of the wealth accumulated in one lifetime is smaller for both smaller values of g and larger values of r. Piketty's statement holds in this time frame, but cannot be extended to a framework in which secular trajectories are considered.

2.2 Alternative Models with Various Categories of Economic Agents

The finding that Piketty's interpretation of a tendency toward rising wealth inequality is unconvincing does not prove that such a tendency does

7. As shown in equation 2, g must be larger than sr, and $g - sr$ is necessarily larger than zero.

not exist in capitalism and that it would be impossible to provide an alternative framework accounting for this inclination. Piketty himself points to two subsidiary mechanisms that might combined their effects to the basic mechanism he puts forward, and add to its effects.

In our earlier study, we denoted as a "simple line of reasoning" the observation that the best accommodated fractions of the population save more than ordinary people and have access to better returns. The present section introduces such models based, respectively, on the two properties above, respectively larger saving rates and larger rates of returns. There is no surprise in the finding that one category of economic agent grows faster and would, asymptotically, become the single owner. The two frameworks can obviously be combined. Concerning saving rates, the two first sections below, assume that only capitalists save (as in Section 1.1.5). A third section relaxes this assumption in the model with distinct saving rates, as wage earners also save but with lower rates of saving.

2.2.1 Capitalists with Distinct Saving Rates

In this first model, the two categories of capitalists benefit from the same rate of return on their investments, but their saving rates, s^1 and s^2, are distinct, with $s^1 > s^2$. (Wage earners do not save.) The two stocks of capital are K^1 and K^2, with the auxiliary notation $x = K^2/K^1$.

Since the two groups of capitalists have unequal saving rates, both the average saving rate and the growth rate depend on x. Aggregate savings (equal to real investment) are: $S = s^1\Pi^1 + s^2\Pi^2 = r(s^1K^1 + s^2K^2)$. The growth rate is:

$$g = \frac{I}{K} = \frac{S}{K} = r\frac{s^1K^1 + s^2K^2}{K^1 + K^2} = r\frac{s^1 + s^2x}{1 + x} \tag{16}$$

The capital stock of the first category of capitalists grows at a rate, $\tau^1 = s^1r$, larger than g, while the stock of the other group grows at a rate, $\tau^2 = s^2r$, smaller than g. The ratio, x, of the two capital stocks decreases: $x_t = x_{t-1}\frac{1+\tau^2}{1+\tau^1}$, that is, $x_t = x_0\left(\frac{1+\tau^2}{1+\tau^1}\right)^t$. Wealth inequality between the two categories of capitalists increases indefinitely. The growth rate, g, rises from $g_0 = r\frac{s^1 + s^2x_0}{1 + x_0}$ for $t = 0$, up to s^1r for $t = \infty$, that is, up to τ^1, the growth rate of the stock of capital of the first group. As is always the case in such models, the capital of the first category of capitalists, gradually becoming the owners of the entire stock of capital, grows asymptotically at the same rate as output.

2.2.2 Capitalists with Distinct Rates of Return

This section considers two categories of capitalists with distinct rates of returns such as the holders of, respectively, stock shares and bonds of corporations.

In this second model, the saving rates of the two categories of capitalists (holding, respectively, K^1 and K^2, with $x = K^2/K^1$) are the same. (As in the previous model, wage earners do not save.) The two rates of return, r^1 and r^2, are distinct, with a given ratio, $r^2/r^1 = \mu$ between the two, and $r^1 > r^2$ or $\mu < 1$. These two rates of return cannot be determined arbitrarily, since they proceed from the payment of the total profits. One necessarily has: $r^1K^1 + r^2K^2 = rK$. Thus, one of the two rates of return must be smaller than r, and the other larger: $r^2 < r < r^1$. The two rates of returns are:

$$r^1 = r\frac{1+x}{1+\mu x} \quad \text{and} \quad r^2 = \mu r^1 \tag{17}$$

Output grows at a rate $g = sr$ (since the two categories of capitalists have the same saving rate). The capital of the first group increases at a rate $\tau^1 = sr^1 > g$, that is, faster than output. The capital of the second group grows at a rate $\tau^2 = sr^2 < g$, that is, slower than output. The ratio, x, between the two capital stocks diminishes:

$$x_t = x_{t-1}\frac{1+\tau^2}{1+\tau^1} = x_{t-1}\frac{1+\mu r\dfrac{1+x_{t-1}}{1+\mu x_{t-1}}}{1+r\dfrac{1+x_{t-1}}{1+\mu x_{t-1}}} \tag{18}$$

The asymptotic value of x is 0. The first category of capitalists will gradually own the entire stock of capital, and its rate of return will decrease and converge to the rate of return of corporations. The wealth of this group may increase faster than output during the period of establishment of its dominance, but not asymptotically.

2.2.3 Large and Small Savers

In this third model, the framework is similar to the above, and the notation is basically conserved. We distinguish between two categories of savers: (1) large capitalists; and (2) other savers, a group in which smaller capitalists and wage earners are jointly involved. Compared to the two previous models, the assumption that only capitalists save is removed. This model is closer to Piketty's framework, in which the saving rate is defined in relation to national income. While the previous models were based on the distinction of two categories of capitalists, the distinction is now made concerning two categories of households.

As in Section 2.2.1, the two groups of agents differ concerning their saving rates, with a larger saving rate s^1 in the first group. The large capitalists receive profits, Π^1, on their capital, K^1, as determined by the rate of return: $\Pi^1 = rK^1$. The other category (smaller capitalists and wage earners) receives the remainder, $Y - rK^1$, as profits and wages. (One can check that this income is positive: $r = \pi P_K < P_K$ and $K_1 < K$. Thus, $rK_1 < P_K K = Y$.)

Total savings are:

$$S = S^1 + S^2 = s^1\Pi^1 + s^2(Y - rK^1) = \\ s^1rK^1 + s^2(Y - rK^1) = (s^1 - s^2)rK^1 + s^2Y$$

The average saving rate is:

$$s = \frac{S}{Y} = \frac{(s^1 - s^2)rK^1 + s^2Y}{Y} = (s^1 - s^2)\frac{\pi}{1+x} + s^2 \quad \text{with } x = \frac{K^2}{K^1} \tag{19}$$

The growth rate is $g = sP_K$, that is:

$$g = \left((s^1 - s^2)\frac{\pi}{1+x} + s^2\right) P_K \tag{20}$$

Growth rates differ: (1) Total output grows at a rate g; (2) The wealth of the large capitalists grows at a rate $\tau^1 = S^1/K^1$; and (3) The wealth of the rest of the population grows at a rate $\tau^2 = S^2/K^2$. These two latter rates can be easily determined:

$$\tau^1 = s^1\pi P_K \qquad \text{and} \qquad \tau^2 = s^2(\pi + \omega\frac{1+x}{x})P_K \tag{21}$$

In the expression of τ^2, the first term, $s^2\pi P_K$, accounts for the savings of the smaller capitalists, and the second term, $s^2\omega\ (1+x)/x\ P_K$, for the savings of wage earners. More specifically, $s^2\omega\ 1/x\ P_K$ accounts for the savings that proceed from the wages of wage earners working for the large capitalists, and $s^2\omega\ P_K$, for the savings of the wage earners working for the smaller capitalists.) One can check that g is equal to the average of τ^1 and τ^2:

$$g = \frac{\tau^1K^1 + \tau^2K^2}{K^1 + K^2}$$

Since $x = K^2/K^1$, and $K^i_t = (1+\tau^i)K^i_{t-1}$ (for $i = 1, 2$), one has: $x_t = x_{t-1}\frac{1+\tau^2}{1+\tau^1}$. The growth rate of the capital of large capitalists is constant (Equation 21) and τ^2 is a function of x. The following recursion is obtained:

$$x_t = x_{t-1}\frac{1 + s^2\left(1 + \frac{\omega}{x_{t-1}}\right)P_K}{1 + s^1\pi P_K}$$

An equilibrium value of x exists that ensures that the two capital stocks grow at the same rate:

$$\tau^1 = \tau^2 \qquad \text{if} \qquad x = x^* = \frac{s^2\omega}{s^1\pi - s^2}$$

Two situations can follow depending on the sign of $s^1\pi - s^2$:

1. If $s^1\pi - s^2 < 0$, there is no equilibrium. In this case, independently of the value of x, the relationship $\tau^2 > \tau^1$ always holds. The capital of the large capitalists will become gradually negligible compared to the capital of other economic agents (since their savings are too low).

2. If $s^1\pi - s^2 > 0$, an equilibrium exists, $x^* > 0$. This equilibrium is stable. If $x < x^*$, one has $\tau^2 > \tau^1$ and x rises over time. If $x > x^*$, one has $\tau^2 < \tau^1$ and x diminishes. The absence of convergence toward $x^* = 0$, as in the previous models, follows from the fact that the growth of large capitalists entails the growth at a similar speed of the wages paid by the sector of the economy these capitalists own. This latter development, in turn, entails the rise at the same rate of the savings, $s^2\omega\ 1/x\ P_K$, of the corresponding wage earners. To sum up, the wealth of large capitalists grows faster than the wealth of other agents if the ratio of their capital to the wealth of other agents is high enough. Asymptotically, all variables grow at the same rate than output, as in the previous models.

2.3 A Rising Tendency toward Wealth Inequality?

Overall, considering jointly the results in the three above frameworks, it appears that the existence of a tendency toward the gradual concentration of wealth in capitalism is not unlikely. This property, actually, matches quite well the basic features of this mode of production. In our earlier article, we mentioned that the prevalence of such a tendency is reminiscent of Marx's views concerning the concentration of capital. The tendency is, however, subject to a number of conditions that might lead to distinguish various periods of time and countries in empirical investigation.

The conclusions in the two first frameworks above are intuitive. Among capitalists, various subcategories may save more and benefit from higher returns, in particular at the top. The conclusions reached in the third model also show, however, that the capability to save and accumulate of other less accommodated fractions of the population may counteract this basic tendency.

More research, both theoretical and empirical investigation, would be required to determine to what extent this model could account for empirical observations. One can first think of the impact of managers accumulating wealth to some extent, and the gradual formation, at a lower level of income, of a "patrimonial middle class", in Piketty's terminology (p. 346), slowing down (in the United States) or offsetting (in countries like France or Germany), the rise of wealth inequality. In our opinion, in the analysis of the trends of wealth inequality, this type of conditional mechanisms is much more relevant than Piketty's reference to the ratio of the rate of return to growth rates, as well as the corresponding interpretations of recent trends in reference to differences in growth rates (as between the United States and Europe). A second promising field (as in the case considered above in the absence of equilibrium) could be the analysis of the historical process of gradual decline of the wealth of capitalist classes as during the postwar compromise [8], or the establishment a new social order in the wake of the current crisis, in which the interests of capitalist classes would be finally affected.

8. One can think, in particular, of the project of unions in the Sweedish social-democracy to gradually dominate the ownership of capital through the accumulation of workers's savings in various funds.

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