Internet Appendix for
"Declining Labor and Capital Shares"

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Contents

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A Model of the Corporate Sector

In this appendix I present a standard general equilibrium model of monopolistic competition to study the decline in the shares of labor and capital. The model is standard in order to ensure that the results are not due to novel modeling features, but rather are a direct consequence of the measurement of the capital share.

The model allows for changes in technology, preferences, relative prices, and competition. While changes to preferences, technology, and relative prices can cause firms to shift from labor to capital, and as a consequence can cause the labor share to decline at the expense of the capital share, these mechanisms cannot cause a simultaneous decline in the shares of both labor and capital. An decline in competition and increase in markups is necessary to match a simultaneous decline in the shares of labor and capital.

I calibrate the model to the U.S. non-financial corporate sector and show that the decline in competition inferred from the data can quantitatively match the decline in the shares of both labor and capital. Using the calibrated model, I further explore the welfare implications of the decline in competition. Across a range of parameter values, the model finds that the decline in competition has led to large gaps in output (8.2% to 10%), wages (18.8% to 19.4%), and investment (14.1% to 19.8%).

A.1 Setup

Final Goods Producer The corporate sector is made up of a unit measure of firms, each producing a differentiated intermediate good. The final good is produced in perfect competition as a CES aggregate of the intermediate goods

$$Y_t = \left(\int_0^1 y_{i,t}^{\frac{\varepsilon_t - 1}{\varepsilon_t}} di \right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}} \tag{A.1}$$

where $\varepsilon_t > 1$ is the elasticity of substitution between goods. The pure profits of the final goods producer are $P_t^Y Y_t - \int_0^1 p_{i,t} y_{i,t} di$, where P_t^Y is the exogenous price level of output and $p_{i,t}$ is the endogenous price of intermediate good i. The solution to the cost minimization problem, together with the zero pure profit condition of the final goods producer, leads to the following demand function for intermediate good i:

$$D_t\left(p_{i,t}\right) = Y_t\left(\frac{p_{i,t}}{P_t^Y}\right)^{-\varepsilon_t} \tag{A.2}$$

Firms Firm i produces intermediate good $y_{i,t}$ using the constant return to scale production function

$$y_{i,t} = f_t\left(k_{i,t}, l_{i,t}\right) \tag{A.3}$$

where $k_{i,t}$ is the amount of capital used in production and $l_{i,t}$ is the amount of labor used in production. In period $t-1$ the firm exchanges one-period nominal bonds for dollars and purchases capital $k_{i,t}$ at the nominal price P_{t-1}^K. In period t the firm hires labor in a competitive spot market at the nominal wage rate w_t and produces good $y_{i,t}$, which is sold at price $p_{i,t}(y)$. After production the firm pays the face value of its debt and sells the undepreciated capital at the nominal price P_t^K. The firm's nominal pure profits are

$$\begin{aligned}
\pi_{i,t} &= \max_{k_{i,t},l_{i,t}} p_{i,t}y_{i,t} - \left(1+i_t\right)P_{t-1}^K k_{i,t} - w_t l_{i,t} + \left(1-\delta_t\right)P_t^K k_{i,t} \\
&= \max_{k_{i,t},l_{i,t}} p_{i,t}y_{i,t} - R_t P_{t-1}^K k_{i,t} - w_t l_{i,t}
\end{aligned} \tag{A.4}$$

where $R_t = i_t - \left(1-\delta_t\right)\frac{P_t^K - P_{t-1}^K}{P_{t-1}^K} + \delta_t$ is the required rate of return on capital.

The pure profit maximization problem of the firm determines the demand for labor and capital inputs, as well as pure profits, as a function of the current period nominal interest rate, the current period nominal wage rate, and aggregate output. The first-order condition for capital is $p_{i,t}\frac{\partial f}{\partial k} = \mu_t R_t P_{t-1}^K$, where $\mu_t = \frac{\varepsilon_t}{\varepsilon_t - 1}$ is the equilibrium markup over marginal cost. Similarly, the first-order condition for labor is $p_{i,t}\frac{\partial f}{\partial l} = \mu_t w_t$. Integrating demand across firms determines the corporate sector demand for labor and capital inputs, as well as pure profits, as a function of the nominal interest rate, the nominal wage rate, and aggregate output.

Households A representative household is infinitely lived and has preferences over its consumption $\{C_t\}$ and its labor $\{L_t\}$ that are represented by the utility function

$$\sum_t \beta^t U(C_t, L_t) \tag{A.5}$$

The economy has a single savings vehicle in the form of a nominal bond: investment of 1 dollar in period t pays $1+i_{t+1}$ dollars in period $t+1$. In addition to labor income and interest on savings, the household receives the pure profits of the corporate sector. The household chooses a sequence for consumption $\{C_t\}$ and labor $\{L_t\}$ to maximize utility subject to the lifetime budget constraint

$$a_0 + \sum_t q_t \left[w_t L_t + \Pi_t\right] = \sum_t q_t P_t^Y C_t \tag{A.6}$$

where a_0 is the initial nominal wealth of the household, $q_t = \prod_{s \leq t} (1+i_s)^{-1}$ is the date zero price of a dollar in period t, w_t is the nominal wage in period t, Π_t are nominal corporate pure profits in period t, and P_t^Y is the price of a unit of output in period t.

The utility maximization problem of the household determines the supply of labor and nominal household wealth as a function of the path of nominal interest rates, the path of nominal wage rates, and the net present value of nominal corporate pure profits. The inter-temporal first-order condition of the household [Euler equation] is $1 = \beta\left(1+i_{t+1}\right)\left(1+\frac{P_{t+1}^Y - P_t^Y}{P_t^Y}\right)^{-1}\frac{U_c(C_{t+1},L_{t+1})}{U_c(C_t,L_t)}$ and the intra-temporal first-order condition [MRS] is $U_l(C_t, L_t) = -\frac{w_t}{P_t^Y} U_c(C_t, L_t)$. The nominal wealth of the household follows the path

$$a_{t+1} = (1+i_t)\, a_t + w_t L_t + \Pi_t - P_t^Y C_t \tag{A.7}$$

Capital Creation I assume that all agents in the model have free access to a constant returns to scale technology that converts output into capital at a ratio of $1:\kappa_t$. I further assume that this technology is fully reversible.[1] Arbitrage implies that, in period t, κ_t units of capital must have the

[1] Without this assumption, the relative price of capital is pinned down so long as investment is positive. In the data, investment in each asset is positive in each period. Moreover, the data show no substantial movement in the relative price of capital over the sample period.

same market value as 1 unit of output. This pins down the relative price of capital

$$\frac{P_t^K}{P_t^Y} = \kappa_t^{-1} \tag{A.8}$$

A.2 Equilibrium

In equilibrium three markets will need to clear: the labor market, the capital market, and the market for consumption goods. The labor market clearing condition equates the household supply of labor with the corporate sector demand for labor. The capital market clearing condition equates the nominal value of household savings a_{t+1} with the nominal value of the corporate sector demand for capital $P_t^K K_{t+1}$. The aggregate resource constraint of the economy, measured in nominal dollars, can be written as

$$P_t^Y Y_t = P_t^Y C_t + P_t^K \left[K_{t+1} - (1-\delta) K_t\right] \tag{A.9}$$

By Walras's law, the aggregate resource constraint of the economy holds if the labor and capital markets clear and the households are on their budget constraint. An equilibrium[2] is a vector of prices $(i_t^*, w_t^*)_{t\in\mathbb{N}}$ that satisfy the aggregate resource constraint and clear all markets in all periods. Since all firms face the same factor costs and produce using the same technology, in equilibrium[3] they produce the same quantity of output $y_t = Y_t$ and sell this output at the same per-unit price $p_{i,t} = P_t^Y$.

A.3 Calibration

I calibrate the model to the U.S. non-financial corporate sector and show that a simultaneous decline in the real interest rate and decline in competition can quantitatively match the decline in the shares of both labor and capital. In addition, I calculate the gaps in output, investment, and wages due to the decline in competition inferred from the data. Across a range of parameter values,

[2] Firm optimization requires that firms have beliefs over aggregate output Y_t and house optimization requires that households have beliefs over corporate pure profits Π_t. Equilibrium further requires that firm beliefs and household beliefs hold true.

[3] With a constant returns to scale production technology and the specified market structure there is no indeterminacy in the firm's maximization problem. In more general cases, indeterminacy may arise, in which case there can exist non-symmetric equilibria. With appropriate regularity conditions, it is possible to select an equilibrium by assuming that for a given level of pure profits firms will choose to maximize their size.

the model finds that the decline in competition, which is measured as a decline in ε and results in the increase in markups, leads to large gaps in output (8.3% to 10%), wages (18.9% to 19.5%), and investment (14.1% to 19.8%).

Functional Form Specifications I assume that firms produce using a CES production function

$$y_{i,t} = \left(\alpha_K \left(A_{K,t} k_{i,t}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_K)\left(A_{L,t} l_{i,t}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{A.10}$$

where σ is the elasticity of substitution between labor and capital. In equilibrium, aggregate output is a CES aggregate of labor and capital with parameters that are identical to the firm-level production function

$$Y_t = \left(\alpha_K \left(A_{K,t} K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_K)\left(A_{L,t} L_t\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{A.11}$$

The first-order conditions of firm optimization are

$$\alpha_K A_{K,t}^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}} = \mu_t R_t \frac{P_{t-1}^K}{P_t^Y} \tag{A.12}$$

$$(1-\alpha_K) A_{L,t}^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}} = \mu_t \frac{w_t}{P_t^Y} \tag{A.13}$$

where $\mu_t = \frac{\varepsilon_t}{\varepsilon_t - 1}$ is the equilibrium markup. I assume that household preferences over consumption $\{C_t\}$ and labor $\{L_t\}$ are represented by the utility function

$$\sum_t \beta^t \left[\log C_t - \gamma \frac{\theta}{\theta+1} L_t^{\frac{\theta+1}{\theta}}\right] \tag{A.14}$$

The intra-temporal first-order condition [MRS] is $\gamma L_t^{\frac{1}{\theta}} = \frac{w_t}{P_t^Y} C_t^{-\eta}$ and the inter-temporal first-order condition of the household [Euler equation] is $1 = \beta\left(1 + i_{t+1}\right)\left(1 + \frac{P_{t+1}^Y - P_t^Y}{P_t^Y}\right)^{-1}\left(\frac{C_{t+1}}{C_t}\right)^{-\eta}$.

Model Parameter Values The model has two capital parameters: the relative price of capital, which I normalize to 1, and the depreciation rate, which I match to the average depreciation rate of capital in the BEA data. The model has four production parameters: I consider values of the elasticity of substitution between labor and capital σ between 0.4 and 0.7; I calibrate the remaining three parameters (α_K, A_K, A_L) to match the labor share and the capital to output ratio in 1984 and to equate the level of output across the different specifications of the elasticity of substitution. The model has three preference parameters: I calibrate the rate of time preference β to match the real interest rate; I set the Frisch elasticity of labor supply θ to 0.5;[4,5] and I normalize the disutility of labor parameter γ to equate the steady-state supply of labor across the different specifications.

Forcing Variables The equilibrium conditions of the model imply that the cost share of gross value added is equal to the inverse of the markup $\mu_t^{-1} = \frac{w_t L_t + R_t P^K_{t-1} K_t}{P^Y_t Y_t}$. I vary competition (measured as the elasticity of substitution between goods) in order to match an increase in markups from 2.5% in 1984 to 21% in 2014. I assume that at the start of the sample the economy is in a steady state with a markup of 2.5% $\left(\varepsilon = \frac{1.025}{1.025-1}\right)$ and at the end of the sample the economy is in a steady state with a markup of 21% $\left(\varepsilon = \frac{1.21}{1.21-1}\right)$. I vary the rate of time preference in order to match the observed change in the real interest rate. I assume that at the start of the sample the economy is in a steady state with a real interest rate of 8.5% $\left(\beta = 1.085^{-1}\right)$ and at the end of the sample the economy is in a steady state with a real interest rate of 1.25% $\left(\beta = 1.0125^{-1}\right)$.

A.4 Results

I present two sets of model-based counterfactual estimates. The first set of model-based counterfactual estimates, which appear in rows 1–3 of Table A.1, are backward-looking: they ask how the labor share, capital share, and investment rate should have evolved from 1984 to 2014 in response to a decline in competition (the elasticity of substitution between goods) and a decline in the real interest rate. The second set of model-based counterfactual estimates are forward-looking: they ask how output, wages, and investment can be expected to evolve from 2014 onward if competition increases to its 1984 level, but at the same time interest rates remained low. I report all comparative

[4] This value is consistent with both micro and macro estimates of the Frisch elasticity of labor supply. See Shimer (2010) and Chetty (2012) for a discussion of micro and macro estimates of the Frisch elasticity.

[5] In unreported results, I consider values of the Frisch elasticity of labor supply θ between 0.5 and 4. Given the preference and technology specifications of the model, the value of the Frisch elasticity affects the level of output, capital, labor, pure profits, and investment, but does not affect the shares of labor, capital, pure profits, or investment. As a consequence, the choice of Frisch elasticity does not affect the shock to competition needed to match this increase in markups, nor does the choice of Frisch elasticity affect the ability of the shock to match the decline in the shares of labor and capital. The choice of Frisch elasticity does have consequences for the gaps in output and investment: the gaps in output and investment are increasing in the value of the Frisch elasticity. In this sense, Table A.1 reports lower bounds on the gaps in output and investment. Results based on alternative values of the Frisch elasticity are available from the author upon request.

statics for a range of values of the elasticity of substitution between labor and capital σ between 0.4 and 0.7.

Rows 1–3 of Table A.1 present the percentage changes in the labor share, the capital share, and the ratio of investment to output across steady state – all in response to the decline in competition and the decline in the real interest rate. In this counterfactual exercise I vary the degree of competition (the elasticity of substitution between goods) in order to match an increase in markups from 2.5% to 21% and I vary the rate of time preference in order to match the observed change in the real interest rate from 8.5% to 1.25%. The model successfully matches the empirically measured declines in the shares of both labor and capital. Across the range of values of the elasticity of substitution between labor and capital the model predicts a decline in the labor share ranging from 9.5% to 12.2% and a decline in the capital share ranging from 23% to 31%. In the data, the labor share declines by 10.6%. In the main specification of the paper, the capital share declines 22%. The model does a reasonable job of matching investment. The observed increase in investment is 14.1% and the model predicts an increase that ranges from 14.2% to 26.2%.

In this first exercise, competition varies in order to match the change in the share of pure profits. Thus, the counterfactual exercise relies on the measurement of capital costs. Since the shares of labor, capital, and pure profits sum to one, by matching the change in the share of pure profits the model will perfectly match the change in combined shares of labor and capital. At the same time, the shares of labor and capital are free to individually vary: a 15 percentage point increase in the share of pure profits is consistent with both (a) a 20 percentage point decline in the share of labor and a 5 percentage point increase in the share of capital, and (b) a 7.5 percentage point decline in the share of labor and a 7.5 percentage point decline in the share of capital. In this sense the model is successful in matching a free moment of the data.

An alternative exercise can help explain the free moment that the model is able to match. Fix the elasticity of substitution between labor and capital at 0.5 (this matches column 2 of Table A.1). We can calibrate the change in competition to match the change in the labor share. This alternative exercise does not require data on capital costs or pure profits; instead it assumes that the decline in the labor share is the result of a decline in competition. In order to match a decline in the labor share of 10.3%, in addition to the decline in the real interest rate, the economy would need to move from the 1984 steady state with a markup of 2.5% $\left(\varepsilon = \frac{1.025}{1.025-1}\right)$ to a steady state with a markup of 21% $\left(\varepsilon = \frac{1.21}{1.21-1}\right)$. Without using any data on pure profits or capital to discipline the model, the model predicts that this decline in competition will be accompanied by a 28.2% decline in the capital share.

Rows 4–6 of Table A.1 present the gap in output, wages, and investment that are due to the decline in competition. In this counterfactual exercise I vary competition (the elasticity of substitution between goods) in order to decrease markups from 21% back down to 2.5% while holding the rate of time preference constant to match the steady state real interest rate of 1.25%. I refer to the steady state of the economy with a 2.5% markup and 1.25% real interest rate as the potential steady state. For a variable X, I compute the gap in X as $\frac{X-X^*}{X^*}$ where X^* is the value of X in the potential steady state. Across the range of values of the elasticity of substitution between labor and capital the model predicts large gaps in output (8.2% to 10%), wages (18.8% to 19.4%), and investment (14.1% to 19.8%). Said differently, the model predicts large improvements to the economy in response to an increase in competition to its 1984 level: we would see large increase in output (8.9% to 11.1%), investment (16.4% to 24.7%), and wages (23.2% to 24.1%).

Taken together, this evidence suggests that the decline in competition and increase in markups inferred from the data can explain the bulk of the decline in the shares of both labor and capital and that the decline in the shares of labor and capital is an inefficient outcome.

A.5 Discussion

This appendix presented a standard general equilibrium model of monopolistic competition and provided three sets of results. First, a decline in competition is necessary to match a joint decline in the shares of labor and capital. While changes to preferences, technology, and relative prices can cause firms to shift from labor to capital, and as a consequence can cause the labor share to decline at the expense of the capital share, these mechanisms cannot cause a simultaneous decline in the shares of both labor and capital. Second, the decline in competition and increase in markups inferred from the data can explain the bulk of the decline in the shares of both labor and capital that we observe in the data from 1984–2014. Last, the model suggests that the decline in competition inferred from the data causes large gaps in output, wages, and investment.

The contribution of a decline in competition to the decline in the labor share depends crucially on our measurement of the capital share. To understand this point it is worth considering three different measurements of the capital share:

1. **Increasing Capital Share.** Consider the case in which the labor share is declining and the capital share increases to fully offset the decline in the labor share. In this case, the model will attribute all of the decline in the labor share to changes in preferences, technology, and relative prices and will attribute none of the decline in the labor share to a decline in competition.[6] If we indirectly infer the capital share as 1 minus the labor share then we are necessarily attributing the decline in the labor share to preferences, technology, and relative prices.

2. **Flat Capital Share.** Consider the case in which the labor share is declining, the capital share does not change, and the pure profit share increases and offsets the decline in the labor share. In this case, the model will attribute part of the decline in the labor share to changes in preferences, technology, and relative prices and will attribute part of the decline in the labor share to a decline in competition. Changes in preferences, technology, and relative prices alone would have caused the capital share to increase; changes in competition alone would have caused the capital share to decline. If we were to measure the capital share under the assumption of a constant required rate of return then we would find that the capital share has remained flat and we would conclude that preferences, technology, and relative prices and competition both contributed substantially to the decline in the labor share. This is precisely the measurement assumption of Karabarbounis and Neiman (2014) and Rognlie (2015). Indeed, based on this measurement assumption Karabarbounis and Neiman (2014) attribute half the decline in the labor share to changes in relative prices and half to an increase in markups.[7]

3. **Declining Capital Share.** Consider the case in which the labor share is declining, the

[6] Further data and modeling assumptions are needed to quantify the separate contributions of preferences, technology, and relative prices.

[7] Table 4 of Karabarbounis and Neiman (2014) presents a specification in which markups increase and relative prices remain constant. In this specification, the shares of both labor and capital decrease. Based on their measurement of the capital share – which assumes a constant required rate of return and finds that the capital share is flat – they conclude that an increase in markups alone is a poor fit for the data.

capital share is declining, and the pure profit share is increasing and offsets the decline in the shares of both labor and capital. In this case, the model will attribute much of the decline to a decline in competition. A precise calibration of the model is needed to determine just how much of the decline in the labor share is due to a decline in competition; the range of calibrations that I considered attribute the bulk of the decline to the decline in competition.

The magnitude of the decline in the capital share is of central importance for understanding why the labor share has declined. Existing research has already documented an increase in the share of pure profits. In addition to the work of Karabarbounis and Neiman (2014) and Rognlie (2015), Hall (2016) documents a growing wedge between the return to capital and the risk-free real interest rate, suggestive of an increase in pure profits. An increase in the share of pure profits is not sufficient to determine the cause of the decline in the share of labor; we need a direct measurement of the pure profit share. Measuring the capital share and using market prices of debt and equity to determine the required rate of return lead us to conclude that (1) the capital share declined (2) a decline in competition inferred from the data can explain the bulk of the decline in the shares of both labor and capital that we observe in the data from 1984–2014 (3) the decline in the labor share is accompanied by increasing gaps in output, wages, and investment.

A.6 The Roles of Technology, Preferences, Relative Prices, and Markups

Proposition 1. *When markups are fixed, any decline in the labor share must be offset by an equal increase in the capital share.*

Proof. In equilibrium, a marginal allocation plan of labor across firms $\{dl_{i,t}\}_i$ increases aggregate output by $\int_0^1 \mu_t \frac{w_t}{P_t^Y} dl_{i,t} di = \mu_t \frac{w_t}{P_t^Y} \int_0^1 dl_{i,t} di$. Since the aggregate output response to a marginal allocation plan depends only on the aggregate increase in labor $\left(dL_t = \int_0^1 dl_{i,t} di \right)$, we have a well-defined notion of the aggregate marginal productivity of labor that is equal to $\frac{\partial Y_t}{\partial L_t} = \mu_t \frac{w_t}{P_t^Y}$. Similarly, for any marginal allocation plan of capital across firms we have $\frac{\partial Y_t}{\partial K_t} = \mu_t R_t \frac{P_{t-1}^K}{P_t^Y}$. Rearranging these equations we have the following expressions for the labor and capital shares of gross value added

$$S_t^L = \mu_t^{-1} \times \frac{\partial \log Y_t}{\partial \log L_t} \tag{A.15}$$

$$S_t^K = \mu_t^{-1} \times \frac{\partial \log Y_t}{\partial \log K_t} \tag{A.16}$$

Summing across the shares of labor and capital we have

$$S_t^K + S_t^L = \mu_t^{-1} \times \underbrace{\left(\frac{\partial \log Y_t}{\partial \log L_t} + \frac{\partial \log Y_t}{\partial \log K_t} \right)}_{\text{scale of production=1}} \tag{A.17}$$

The combined shares of labor and capital are a function of markups alone. Thus, holding markups fixed, any decline in the labor share must be offset by an equal increase in the capital share. □

The proof of the proposition relies on firm optimization. The proposition holds in equilibrium, not just in steady state. The proof of the proposition is under an assumption of constant returns to scale; more generally, if production is homogeneous of degree γ then the combined shares of labor and capital are equal to $S_t^K + S_t^L = \mu_t^{-1} \times \gamma$.

No assumptions of household behavior, firm ownership, or the functional form of the production function are needed. The degree of generality of this proposition allows us to evaluate several alternative explanations for the decline in the labor share. In all of the following cases, the capital share needs to adjust to perfectly offset the decline in the labor share. Since the data show a decline in the capital share, these explanations alone are unable to match the data.

1. **TFP.** Consider the production function

$$f_t(k, l) = A_t f(k, l)$$

where f is homogeneous of degree 1 (or any other constant degree) in capital and labor. A decline in productivity A_t or a decline in the growth rate of productivity does not affect the combined shares of labor and capital.

2. **Capital Biased Technological Change.** Consider the production function

$$f_t(k, l) = \left(\alpha_K (A_{K,t} k)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_K)(A_{L,t} l)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

Biased technological change, which can be measured as a change to the ratio $\frac{A_{K,t}}{A_{L,t}}$, can cause firms to shift from one input to the other, but does not affect the combined shares of labor and capital.

3. **Relative Prices.** A decline in the price of capital, whether due to improvements in the technology of capital creating or due to an increase in the supply of capital, reduces the price of capital relative to labor. With appropriate assumptions on the elasticity of substitution between labor and capital, the decline in the relative price of capital can cause the labor share to decline, but does not affect the combined shares of labor and capital.

Many other explanations can fit into this simple framework, including changes in the supply of labor and heterogeneous labor and capital inputs. With appropriate assumptions, each of these alternative explanations can cause a decline in the labor share, but does not effect the combined shares of labor and capital.[8] In this sense, a decline in competition, which is measured as a decline in ε and results in the increase in markups, is necessary to match a simultaneous decline in the shares of labor and capital.

[8]We can separate the firm's optimization problem into cost minimization and pure profit maximization. The first-order condition of cost minimization equates the labor share of costs to the elasticity of output to labor. The alternative explanations discussed above share a common prediction: the decline in the labor share of value added perfectly tracks a decline in the labor share of costs. In the data, the capital share is declining faster than the labor share and as a consequence the labor share of costs is increasing.

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Table A.1: **Model-Based Counterfactuals (Percentage Change Across Steady State)**
σ is the elasticity of substitution between labor and capital. Rows 1–3 present steady-state changes in response to the increase in markups and the decline in the real interest rate. Rows 4–6 present the gaps in output, wages, and investment that are due to the increase in markups. See Section A.4 for further details.

	$\sigma = 0.4$	$\sigma = 0.5$	$\sigma = 0.6$	$\sigma = 0.7$	Data
Labor share	-9.5	-10.3	-11.2	-12.2	-10.6
Capital share	-30.5	-28.2	-25.7	-23.2	-22.0
Investment-to-output	14.2	18.1	22.1	26.2	14.1
Output gap	-8.2	-8.7	-9.3	-10.0	
Wage gap	-18.8	-19.0	-19.2	-19.4	
Investment gap	-14.1	-16.0	-17.9	-19.8	

B Model with Adjustment Costs

In this appendix, I incorporate quadratic adjustment costs into a baseline model of monopolistic competition. In the model, firms own the capital stock and choose a path of investment that maximizes their market value. In the model, I mimic the empirical measurement of capital costs and pure profits. I calculate capital costs as $(r + \delta) \times K$ and I calculate pure profits as gross operating surplus less capital costs.

For a wide range of adjustment cost parameters, I compute the unconditional means and standard deviations of the labor, capital, and pure profit shares. I find that these unconditional means are insensitive to the adjustment cost parameter. In this sense, a change to the adjustment cost parameter should not result in a change to the long-run level of the labor share or pure profit share.

In addition, for different values of the adjustment cost parameter I compute the pairwise correlations of the labor, capital, and pure profit shares. For any positive adjustment cost parameter, the labor and pure profit shares are procyclical and positively correlated. Models with higher values of the adjustment cost parameter feature higher correlations between the labor and pure profit shares. These results suggest that a path of shocks that lead to higher measured pure profits should also lead to a higher labor share.

This appendix is organized as follows. Section B.1 presents the model, Section B.2 calibrates the model, Section B.3 describes the measurement of capital costs and pure profits, and Section B.4 presents the numerical results.

B.1 Setup

Households A representative household is infinitely lived and has preferences over its consumption $\{C_t\}$ and its labor $\{L_t\}$ that are represented by the utility function

$$\mathbb{E}\left[\sum_t \beta^t u\left(C_t, L_t\right)\right] \tag{B.1}$$

The economy has a single savings vehicle in the form of equity in the corporate sector. Households are endowed with all equity shares. Equity shares are in zero net supply and the number of shares is normalized to 1. The household chooses a sequence for consumption $\{C_t\}$, labor $\{L_t\}$, and equity shares $\{S_t\}$ to maximize utility subject to the sequence of budget constraints

$$P_t^Y C_t + (S_t - S_{t-1}) V_t = w_t L_t + S_{t-1} Div_t \tag{B.2}$$

where Div_t are total corporate dividends, V_t is the end-of-period nominal value of one share of equity, and $(S_t - S_{t-1}) V_t$ are nominal purchases of equity shares.

Final Goods Producer The corporate sector is made up of a unit measure of firms, each producing a differentiated intermediate good. The final good is produced in perfect competition as a CES aggregate of the intermediate goods

$$Y_t = \left(\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{B.3}$$

where $\varepsilon > 1$ is the elasticity of substitution between goods. The pure profits of the final goods producer are $P_t^Y Y_t - \int_0^1 p_{i,t} y_{i,t} di$, where P_t^Y is the exogenous price level of output and $p_{i,t}$ is the endogenous price of intermediate good i. The solution to the cost minimization problem, together with the zero pure profit condition of the final goods producer, leads to the following demand function for intermediate good i:

$$D_t(p_{i,t}) = Y_t \left(\frac{p_{i,t}}{P_t^Y} \right)^{-\varepsilon} \tag{B.4}$$

Firms Firm i produces intermediate good $y_{i,t}$ using the constant return to scale production function

$$y_{i,t} = f_t(k_{i,t}, l_{i,t}) \tag{B.5}$$

where $k_{i,t}$ is the amount of capital used in production, and $l_{i,t}$ is the amount of labor used in production. The firm owns a stock of capital that evolves according to the law of motion

$$P_t^K k_{i,t+1} = (1-\delta) P_t^K k_{i,t} + P_t^K I_{i,t} - \frac{\psi}{2} \left(\frac{I_{i,t}}{k_{i,t}} - \delta \right)^2 P_t^K k_{i,t} \tag{B.6}$$

where P_t^K is the nominal price of capital, ψ is the adjustment cost parameter, and $\frac{\psi}{2}\left(\frac{I_{i,t}}{k_{i,t}}-\delta\right)^2 P_t^K k_{i,t}$ is the nominal adjustment cost. In period t the firm hires labor in a competitive spot market at the nominal wage rate w_t and produces good $y_{i,t}$, which is sold at price $p_{i,t}(y)$. After production the firm chooses investment and pays dividends

$$Div_{i,t} = p_{i,t}y_{i,t} - w_t l_{i,t} + P_t^K I_{i,t} \tag{B.7}$$

where $p_{i,t}y_{i,t}$ are the firm's revenues, $w_t l_{i,t}$ are the firm's labor costs, and $P_t^K I_{i,t}$ is gross nominal investment. The firm chooses a sequence for labor $\{l_{i,t}\}$, capital $\{k_{i,t}\}$, and investment $\{I_{i,t}\}$ to maximize the net present value of dividends

$$V_{i,t} = \max \mathbb{E}_t \left[\sum_{k\geq 0} \beta^k \frac{\Lambda_{t+k}}{\Lambda_t} Div_{i+t}\right] \tag{B.8}$$

where $\beta^k \frac{\Lambda_{t+k}}{\Lambda_t}$ is the household's marginal rate of substitution.

Capital Creation I assume that all agents in the model have free access to a constant returns to scale technology that converts output into capital at a ratio of $1 : \kappa_t$. I further assume that this technology is fully reversible. Arbitrage implies that, in period t, κ_t units of capital must have the same market value as 1 unit of output. This pins down the relative price of capital

$$\frac{P_t^K}{P_t^Y} = \kappa_t^{-1} \tag{B.9}$$

B.2 Equilibrium

In equilibrium three markets will need to clear: the labor market, the market for consumption goods, and the market for firm equity. The aggregate resource constraint requires that nominal output is equal to the sum of nominal consumption and nominal gross investment.

B.3 Calibration

Functional Form Assumptions I assume that firms produce using a CES production function

$$y_{i,t} = Z_t A_H \left(\alpha_K \left(k_{i,t} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_K) \left(l_{i,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{B.10}$$

where σ is the elasticity of substitution between labor and capital. In equilibrium, aggregate output is a CES aggregate of labor and capital with parameters that are identical to the firm-level production function

$$Y_t = Z_t A_H \left(\alpha_K \left(K_t \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_K) \left(L_t \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{B.11}$$

I assume that household flow utility from consumption C_t and labor L_t is represented by the utility function

$$u \left(C_t, L_t \right) = \log C_t - \gamma \frac{\theta}{\theta + 1} L_t^{\frac{\theta+1}{\theta}} \tag{B.12}$$

I assume that $\log Z_t$ follows an AR(1) process

$$\log Z_t = \rho \log Z_{t-1} + \eta_t \tag{B.13}$$

where the η_t are iid.

Normalization, Parameter Values, and Shock Distribution I normalize the price of the final good to 1 $\left(P_t^Y = 1\right)$. I set the depreciation rate of capital to 0.1 and the relative price of capital to 1 $\left(P_t^K = 1; \delta = 0.1\right)$. I set the rate of time preference to 0.95 and I set θ to 2, and I calibrate γ to match steady-state labor of $\frac{1}{3}$. I set the elasticity of substitution between labor and capital to 0.6, I calibrate α_K to match a steady-state labor share of 0.712, and I choose A_H to normalize steady-state output to 1. I set the demand elasticity to match markups of 2.4% $\left(\varepsilon = \frac{1.024}{1.024-1}\right)$. Last, I set the AR(1) coefficient ρ to 0.95 and assume that the shocks η_t are iid normal with mean zero and standard deviation of 0.1.

B.4 Measuring Capital Costs and Pure Profits

Using the model I construct the equivalent of the empirical estimation of capital costs or pure profits. In the previous section I normalized the price of output and capital to 1 and I drop reference to these in the equations that follow.

- I calculate the firm's cost of capital r_t as expected return on firm equity at the end of period $t-1$.
- The required rate of return on capital is $R_t = r_t + \delta$.
- Capital costs are $R_t K_t$ and pure profits are $\Pi_t = Y_t - w_t L_t - R_t K_t$.
- The capital share is the ratio of capital costs to output and the pure profit share is the ratio of pure profits to output.

B.5 Solution and Model Statistics

I use the gEcon package for R to compute the model steady state and the first-order perturbation solution of the stochastic model. The model steady state does not depend on the adjustment cost parameter ψ. For each value of the adjustment cost parameter ψ, I solve the model and simulate 10,000 random paths, each of length 500 (I simulate paths of length 1,000 and burn the first 500 observations). In this exercise, the adjustment cost parameter ψ varies[9] from 0 to 40.

Unconditional Means and Standard Deviations Figure B.1 presents the unconditional means and standard deviations of the labor share, capital share, and pure profit share for a range of values of the adjustment cost parameter ψ. The unconditional mean of each variable is measured as the percentage point deviation from its value in the common deterministic steady state. The height of each bar represents the unconditional standard deviation.

An increase in the adjustment cost parameter ψ has virtually no effect on the unconditional means of the labor share, capital share, and pure profit share. In all cases, the point estimate for the unconditional mean is near zero. An increase in the adjustment cost parameter ψ does have implications for the standard deviation of the output shares. An increase in the adjustment cost

[9] This range includes implausibly high values of adjustment costs. See Tobin (1981) and Hall (2001) for further details.

parameter reduces the standard deviation of the labor share and increases the standard deviations of the capital and pure profit shares.

Correlations Table B.1 presents the correlations of the labor share, capital share, pure profit share, and $\log(TFP)$ for two particular values of the adjustment cost parameter ψ. Panel A presents correlations for $\psi = 1$ (a low value) and Panel B presents correlations for $\psi = 32$ (a high value). The table shows a positive correlation between the labor share and the pure profit share and that both are procyclical. Furthermore, a higher value of ψ is associated with an increased correlation between the labor share and pure profit share.

Figure B.2 presents the pairwise correlations of the labor share, capital share, and pure profit share for a wide range of values of the adjustment cost parameter ψ. For all positive values of ψ, the correlation between the labor share and the pure profit share is positive and increasing with the value of ψ. For all values of ψ, the correlation between the capital share and the pure profit share is negative and declining with the value of ψ. At high values of ψ the correlation between that capital share and pure profit share falls below 0.9.

References

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Tobin, James. 1981. "Discussion of Summers." *Brookings Papers on Economic Activity*, 1981(1): 132–139.

Figure B.1: **Unconditional Means and Standard Deviations**
The figure shows the unconditional means and standard deviations of the labor share, capital share, and pure profit share for a range of values of the adjustment cost parameter ψ. The unconditional mean of each variable is measured as the percentage point deviation from its value in the common deterministic steady state. The height of each bar represents the unconditional standard deviation. See Section B.5 for further details.

(a) Labor Share

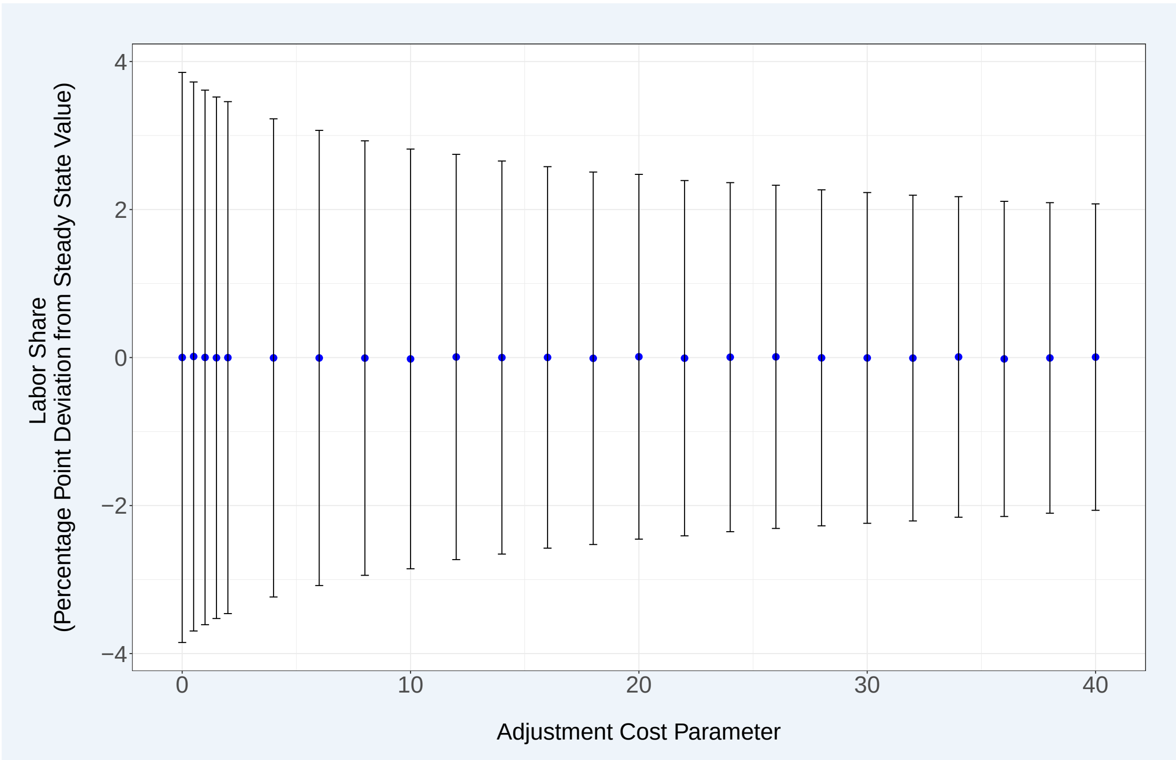
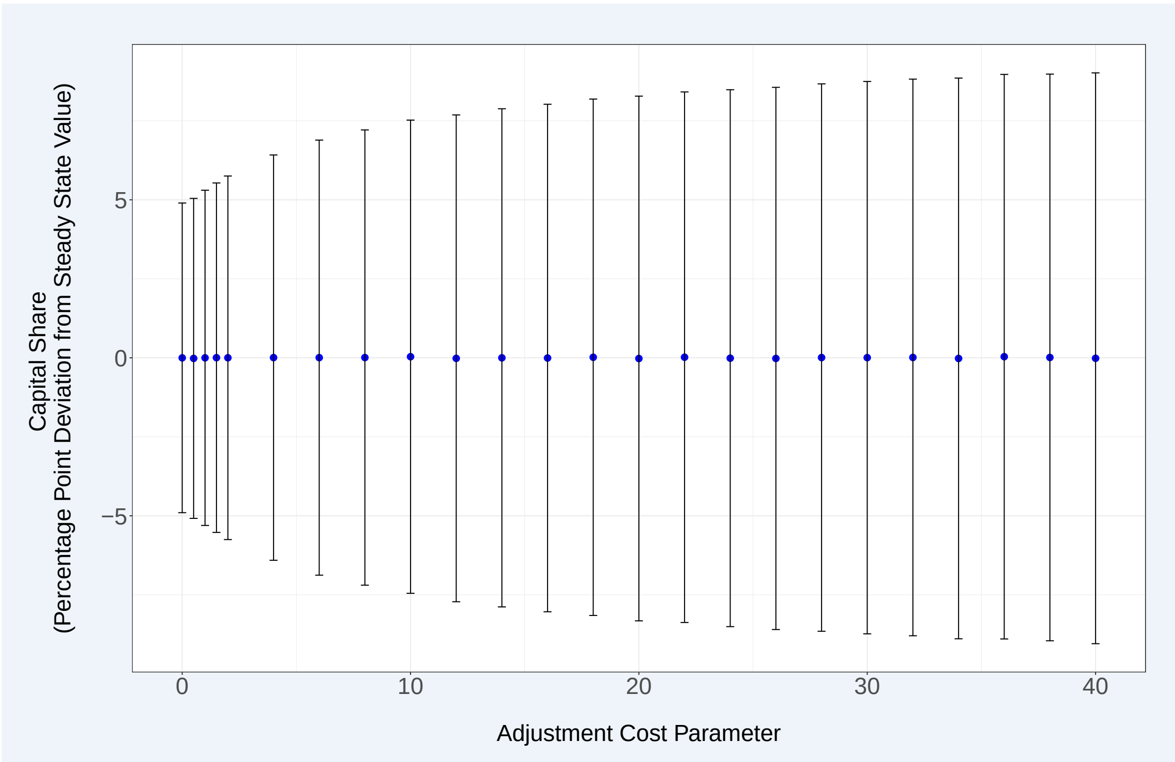


Figure B.1: **Unconditional Means and Standard Deviations** (continued from previous page)

(b) Capital Share



(c) Pure Profit Share

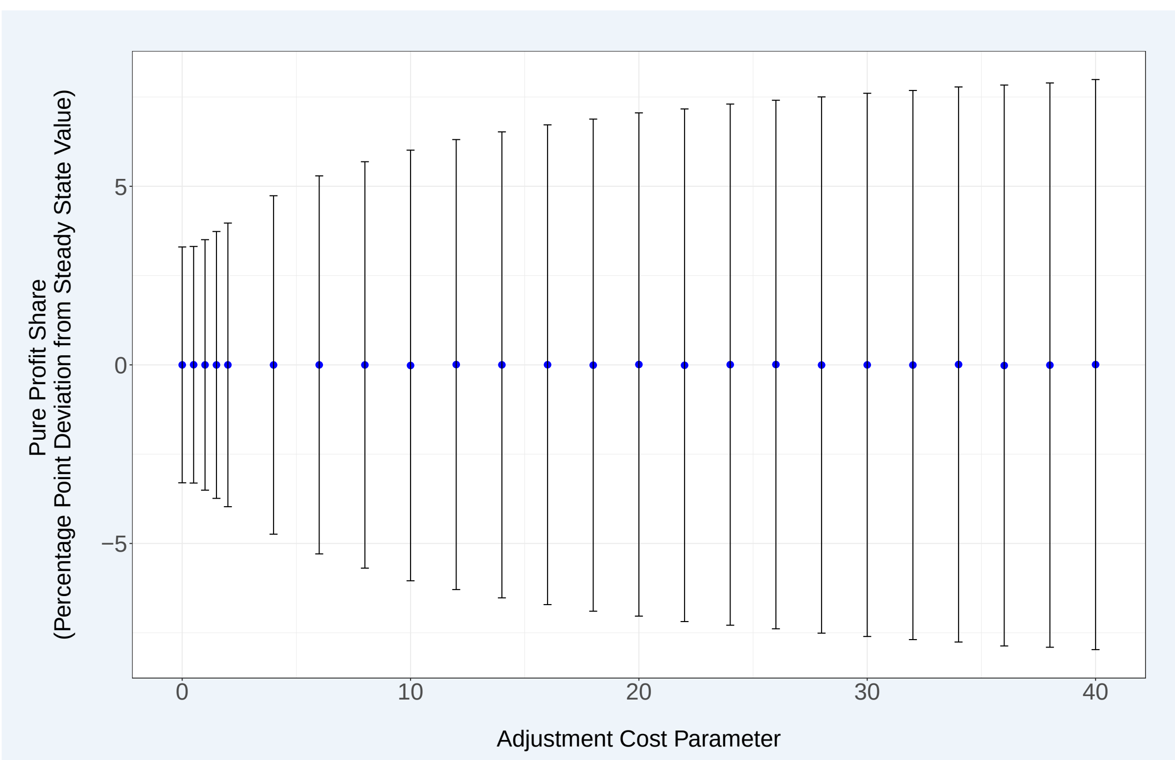


Figure B.2: **Correlations**

The figure shows the pairwise correlations of the labor share, capital share, and pure profit share for a range of values of the adjustment cost parameter ψ. See Section B.5 for further details.

(a) cor(Labor Share, Pure Profit Share)

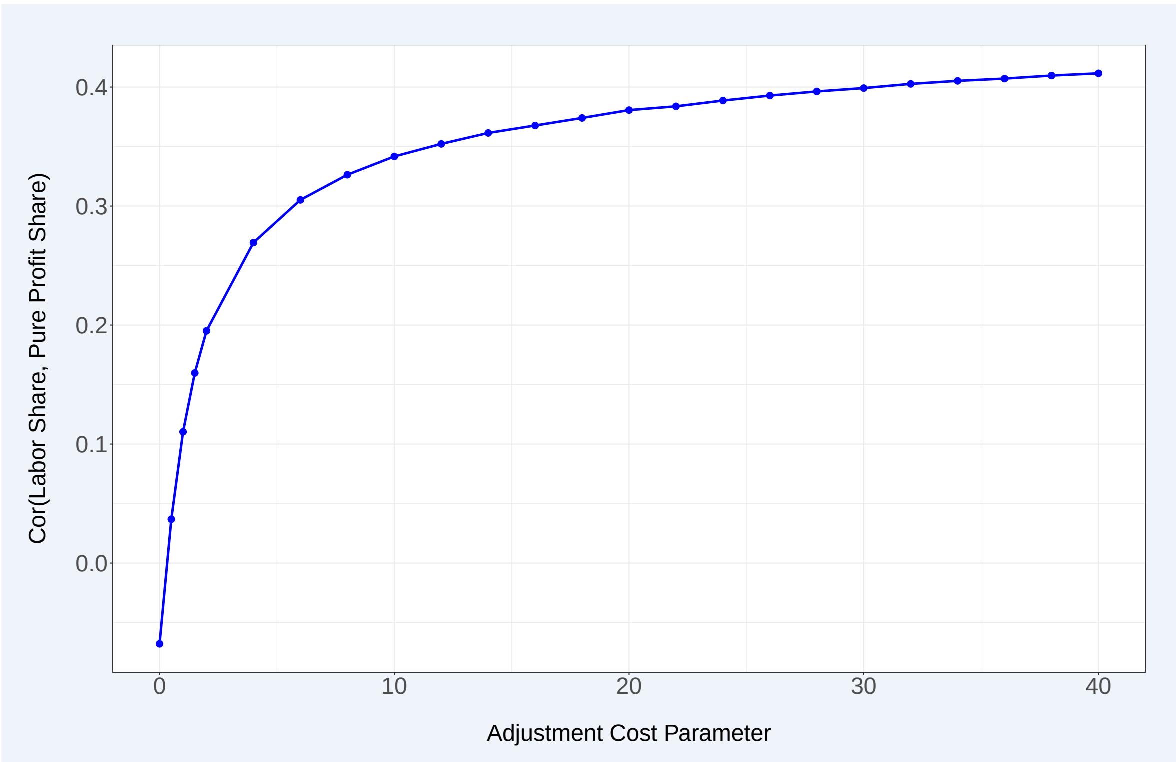
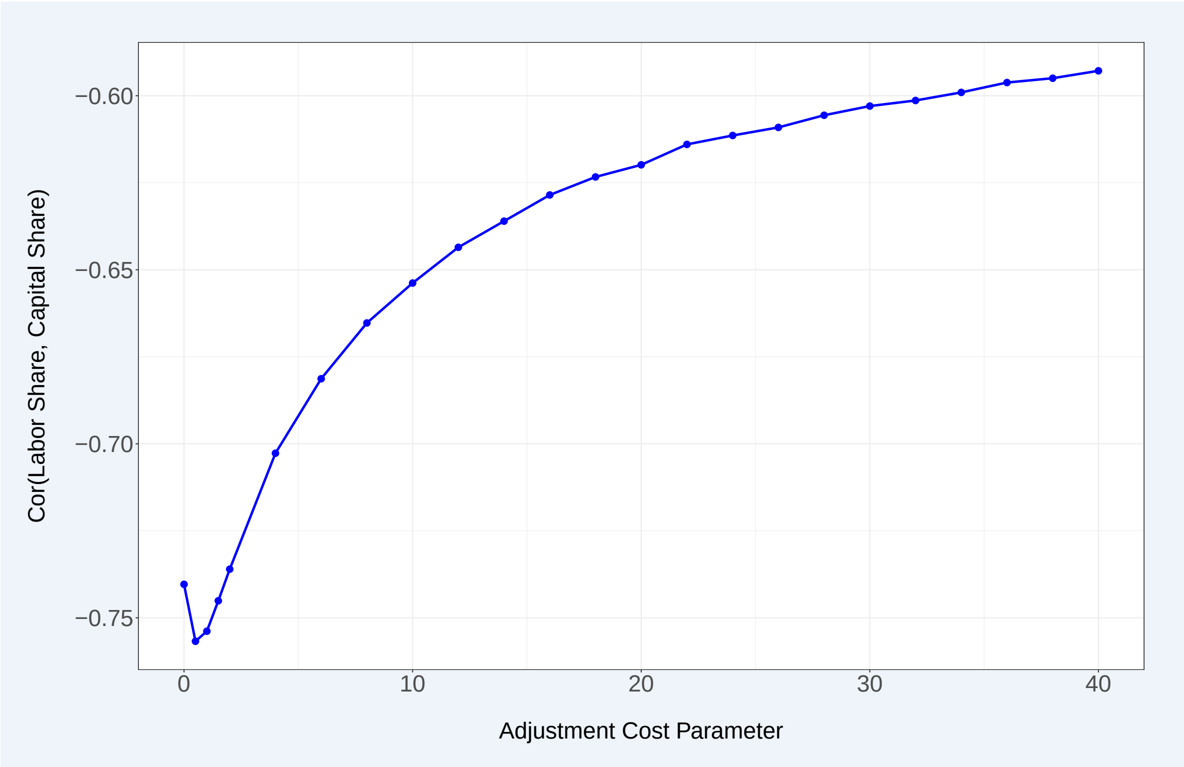


Figure B.2: **Correlations** (continued from previous page)

(b) cor(Labor Share, Capital Share)



(c) cor(Capital Share, Pure Profit Share)

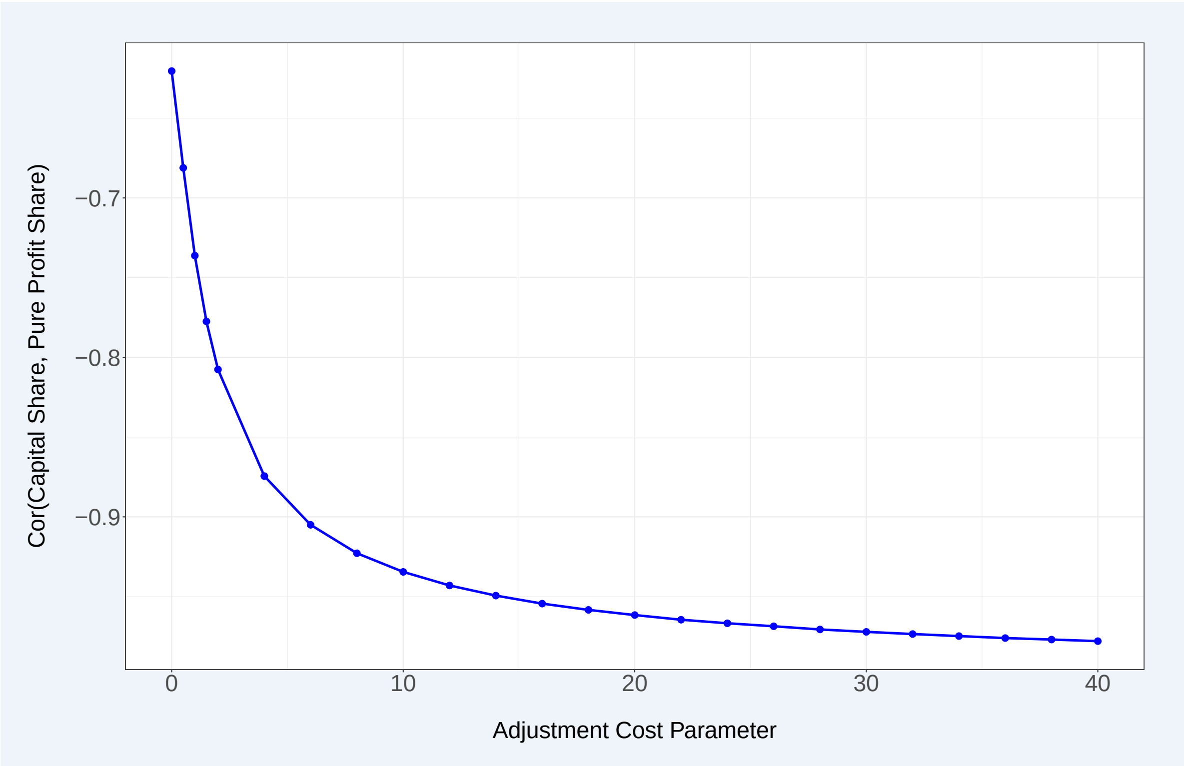


Table B.1: **Correlation Matrix by Adjustment Cost Parameter**

(a) Low Adjustment Costs ($\psi = 1$)

	Labor Share	Capital Share	Pure Profit Share	log(TFP)
Labor Share	1.000	-0.754	0.110	0.842
Capital Share	-0.754	1.000	-0.736	-0.966
Pure Profit Share	0.110	-0.736	1.000	0.594
log(TFP)	0.842	-0.966	0.594	1.000

(b) High Adjustment Costs ($\psi = 32$)

	Labor Share	Capital Share	Pure Profit Share	log(TFP)
Labor Share	1.000	-0.601	0.403	0.741
Capital Share	-0.601	1.000	-0.973	-0.979
Pure Profit Share	0.403	-0.973	1.000	0.910
log(TFP)	0.741	-0.979	0.910	1.000